

# Statistical Models and Data Analysis

Summer term 2017

## Problem Set 3

9.5.2017

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The solutions to this exercise should be ready by 11 am on 17.5.2017. If you have any questions, please send me an email at stemmler@bio.lmu.de. You may submit your exercises by email in one file, but please be legible!

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### 1. (Complex numbers)

(a) Compute the real and imaginary part of the following expressions:

$$i \quad (3+i) - (5+2i) \quad (3+i)(5+2i) \quad (3+i)/(5+2i) \quad (3+i)^2 \quad 4 e^{i\pi/4} .$$

(b) Compute the modulus and argument of the following expressions:

$$i \quad 2 e^{i\pi/2} \quad (2+2i) \quad (\sqrt{3}-i) \quad \ln(-2) \quad \sqrt{-2} \quad (4 e^{i\pi/4})^5 .$$

(c) Find all complex numbers  $z$  that solve the equations

$$z^2 = i \quad z^3 - 1 = 0 \quad z^4 + 2z^2 + 1 = 0 .$$

2. (Complex functions) Consider  $f(z) = z^2$  for  $z = x + iy$ . Let  $u(x, y) = \operatorname{Re}[f(z)]$  be the real part and  $v(x, y) = \operatorname{Im}[f(z)]$  be the imaginary part of  $f(z)$ . Show that, for this  $f(z)$ , the following hold:

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$$
$$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

The symbols  $\partial/\partial x$  and  $\partial/\partial y$  symbolize partial derivatives. For instance,  $\frac{\partial u(x, y)}{\partial y}$  signifies taking the derivative of  $u(x, y)$  with respect to  $y$  while keeping  $x$  constant.

### 3. (Complex numbers and matrices)

Consider the real matrices

$$E = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \text{and} \quad I = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} .$$

Show that for all matrices  $A = x_1 E + y_1 I$  and  $B = x_2 E + y_2 I$ , the following matrices  $C$  can be expressed as  $C = x_3 E + y_3 I$  and give the corresponding expressions for  $x_3$  and  $y_3$ !

$$C = A + B \quad C = AB \quad C = BA \quad C = A^{-1} B \quad C = \exp[A] \exp[-B]$$

Note that the inverse of a 2x2 matrix  $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$  equals

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} ,$$

as long as  $ad - bc \neq 0$ .

The exponential of a matrix  $M$  is defined as

$$\exp [M] = \sum_{n=0}^{\infty} \frac{M^n}{n!}$$

where  $n! = n(n - 1)(n - 2) \cdots 2 \cdot 1$ , which is called the factorial of  $n$ .

Try to interpret these results in terms of complex numbers.

4. (Quaternions) Consider the two complex 2x2 matrices

$$A = \begin{pmatrix} z_1 & \zeta_1 \\ -\bar{\zeta}_1 & \bar{z}_1 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} z_2 & \zeta_2 \\ -\bar{\zeta}_2 & \bar{z}_2 \end{pmatrix} .$$

Show that the matrices

$$C = A + B \quad C = AB \quad C = AB - BA \quad C = A^{-1}$$

have the same shape  $C = \begin{pmatrix} z_3 & \zeta_3 \\ -\bar{\zeta}_3 & \bar{z}_3 \end{pmatrix}$  and give expressions for  $z_3$  and  $\zeta_3$ .

Let's consider the three quaternions

$$I = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} , \quad J = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} , \quad \text{and} \quad K = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} .$$

Show that  $I^2 = J^2 = K^2 = IJK = -1$ .