Statistical Models and Data Analysis

Summer term 2017

Problem Set 5 **24.05.2017**

The solutions to this exercise should be ready by 11 am on 31.5.2017. If you have any questions, please send me an email at stemmler@bio.lmu.de. You may submit your exercises by email in one file, but please be legible!

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1. (Fourier transforms) Compute the Fourier transforms of the following functions:

$$g(x) = \exp\left(-\frac{x^2}{2}\right) \qquad \text{(use the fact that } \int_{-\infty}^{\infty} \exp(-x^2)\, dx = \sqrt{\pi}\text{)}$$

$$\mathrm{sinc}(x) = \frac{\sin(x)}{x} \qquad \text{(rewrite the sine function as the imaginary part of } \exp(ix)\text{)}$$

Plot both the original function and its Fourier transform.

2. (Similarity transformation) Given a function $f(x)$, we can ask what happens to $f(x)$ and its Fourier transform $\mathcal{F}(f(x)) = \widetilde{f}(\omega)$ when we rescale the x-axis by letting $x \to ax$ with $a > 0$.

Using the definition of the Fourier transform, as shown in class, show that

$$\mathcal{F}(f(ax)) = \frac{1}{a}\,\widetilde{f}\left(\frac{s}{a}\right).$$

Use this property and the result from the first exercise to compute the Fourier transform of a Gaussian function $g(x) = \exp(-x^2/(2\sigma^2))$. If σ is large (and the Gaussian is broad), what happens to the Fourier transform?

3. (Fourier transform of an amplitude modulated signal)

Consider the function

$$f(t) = [1 + a\,\sin(\omega_m\, t)]\,\sin(\omega_c\, t)$$

with $0 \leq a \leq 1$.

Compute the Fourier-Transform of $f(t)$. To do so, express the cosine functions in terms of complex exponentials and use the formula for the Fourier representation of the delta function

$$\delta(y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{i}xy}\mathrm{d}x$$

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