Statistical Models and Data Analysis

Summer term 2017

Problem Set 6 **31.05.2017**

The solutions to this exercise should be ready by 11 am on 7.6.2017. If you have any questions, please send me an email at haas@bio.lmu.de. You may submit your exercises by email in one file, but please be legible!

---

1. (Fourier Transform)

(a) Compute the Fourier transform of

$$f(t) = \delta(t-a) + \delta'(t-a)$$

in which $a \in \mathbf{R}$ and δ' is the derivative of the Dirac-δ.

2. (Fourier Transform 2)
Be $\hat{f}(\omega)$ the Fourier transsform of $f(x)$. Compute the back transform $h(x)$ of

$$\hat{h}(\omega) = \frac{1}{2}\left[\hat{f}(\omega-\omega_0) + \hat{f}(\omega+\omega_0)\right] .$$

3. (Fourier Transform 3)
Be

$$\hat{f}(\omega) = \mathrm{i}\,\omega\, \exp\left(-\frac{\omega^2}{2}\right) .$$

the Fourier transform of $f(x)$! Determine the backtransform $f(x)$.
Hint: Express $\mathrm{i}\,\omega$ in the Fourier space as a derivative in the x-space.

4. (Fourier convolution theorem)
Consider the following functions $f(x)$ and $g(x)$:

$$f(x) = \cos(\alpha x)$$

$$g(x) = \begin{cases} \exp(-\frac{x}{\tau}) & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

with $\alpha, \tau \in \mathbf{R}$

(a) Compute the Fourier transforms of the functions $f(x)$ and $g(x)$.

(b) Use $\widehat{f * g}(\omega) = \hat{f}(\omega)\hat{g}(\omega)$ to calculate the Fourier transform of the convolution $(f * g)(x)$.

(c) Compute the inverse Fourier transform of $\widehat{f * g}(\omega)$.

Intermediate solution:
$\hat{f}(\omega) = \pi\{\delta(\omega-\alpha) + \delta(\omega+\alpha)\}$ and $\hat{g}(\omega) = \frac{\tau - i\omega\tau^2}{1+\omega^2\tau^2}$.