Statistical Models and Data Analysis

Summer term 2018 — Prof. Dr. Christian Leibold

Problem Set 2 **4.16.2018**

The solutions to this exercise should be ready by 2 pm on April 23, 2018. If you have any questions, please send me an email at stemmler@bio.lmu.de. You may submit your exercises by email in one file, but please be legible!

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1. (Filtering spike trains) Many neurons emit sequences of stereotypical pulse-like electrical discharges known as spike trains. A mathematical bare-bones description $S(t)$ of the spike train represents it as a sum of delta-functions,

$$S(t) = \sum_{i=1}^{N} \delta(t - t_i),$$

where the t_i denote the times at which the neuron generated an action potential.

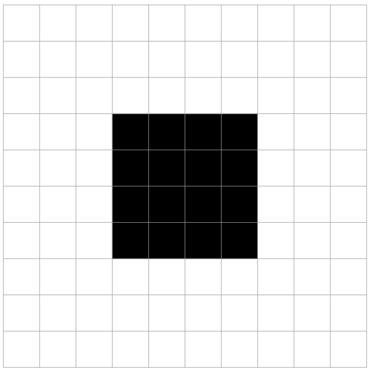
(a) Write down the formula for $S(t)$ for a spike train with four spikes at times $t_i = 2, 4, 5$ and 8 ms and sketch it.

(b) Assume that this signal is filtered either by a Gaussian filter with width $\sigma_f = 1$ ms or a filter that is zero for negative τ and decays exponentially for positive τ with a time-constant $\tau_f = 1$ ms. Write these two filters in mathematical terms, assuming that they are normalized such that the total area under each filter function is unity. For the precise shape of the Gaussian filter, you may use information you can find on the internet or suitable textbooks, for the exponential filter, you should calculate the normalization constant yourself.

(c) Sketch the filtered spike trains in both cases. The resulting curves are often interpreted as smoothed "firing rates" of a neuron. Discuss the qualitative differences between these two cases.

(d) Finally, plot the instantaneous firing rate, defined for every point in time t as the inverse interspike interval between the last spike before t and the next spike after t. What are the most important differences to the findings in (c)?

2. (Convolution) Consider the following 10×10 matrix:



where black represents the value -1 and white represents the value $+1$.

(a) Convolve the matrix with the following stencil: , where, once again, black represents the value -1 and white represents the value $+1$. Specifically, replace each element in the matrix by

$$x_{i,j} \to -x_{i,j} + x_{i,j+1}$$

and assume that any values outside of the bounds of the matrix can be replaced by the value of $+1$. In other words, replace each element in the matrix by the difference between the element just to the right and the element itself.

Sketch the result as a 10×10 matrix. (Recommendation: don't try to write down mathematical formulae to solve this problem).

A stencil is a discrete version of a convolution kernel.

(b) Propose a stencil that will pick out the edges of the square in the 10×10 matrix. You may allow for a shift of the edges by a pixel in either direction.

3. (Integral transforms) Convolutions are integrals of the type

$$(x * y)(t) = \int_0^t x(\tau)y(t-\tau)\ dt$$

You will soon learn about Fourier transforms in the lectures, and how taking the Fourier transform makes convolutions very simple. An integral transform that is closely related to the Fourier transform is called the Laplace transform

$$\mathcal{L}f(s) = \int_0^\infty \exp(-st)f(t)\ dt.$$

(a) Compute the Laplace transform for

$$f(t) = 1 \qquad \text{and} \qquad f(t) = \exp(-at) \quad \text{for } a > 0$$

(b) For a convolution of two functions g and f,

$$\mathcal{L}(g * f)(s) = \mathcal{L}g(s) \cdot \mathcal{L}f(s),$$

so that a convolution becomes a simple multiplication in the integral transform space. (Bonus points for showing why this is true!) Use this convolution property to show that

$$\mathcal{L}(\exp(-at) * \exp(-bt)) = \frac{1}{s+a} \cdot \frac{1}{s+b}$$

(c) Use the fact that

$$\frac{1}{s+a} \cdot \frac{1}{s+b} = \frac{1}{a-b}\left[\frac{1}{s+b} - \frac{1}{s+a}\right]$$

and the result from part (a) to compute the result of convolving $\exp(-at)$ with $\exp(-bt)$.

(d) Plot the result of convolving $\exp(-at)$ with $\exp(-bt)$ as a function of t when you set $a = 1$ and $b = 5$.

In neurobiological terms, you might imagine that a synaptic current has an exponential time-course, but that this is convolved by the membrane time constant to yield an EPSP (excitatory postsynaptic potential). The EPSP time-course would then be given by an expression like the one you just calculated.