Statistical Models and Data Analysis

Summer term 2018

Problem Set 4 **7.5.2018**

The solutions to this exercise should be ready by 2 pm on 14.5.2018. If you have any questions, please send me an email at stemmler@bio.lmu.de. You may submit your exercises by email in one file, but please be legible!

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1. (Orthogonality of sine and cosine functions)

Let us define a "dot product" between two functions $f(t)$ and $g(t)$ as the integral $T^{-1}\int_0^T f(t)g(t)dt$.

(a) Calculate this product for $f(t) = \sin(\frac{2\pi nt}{T})$ and $g(t) = \sin(\frac{2\pi mt}{T})$ where n and m are integer numbers. Perform this calculation without using your knowledge about complex numbers, i.e., use the addition theorems for the sine and cosine. Pay particular attention to the case $n = m$.

(b) Perform the same calculation for $f(t) = \sin(\frac{2\pi nt}{T})$ and $g(t) = \cos(\frac{2\pi mt}{T})$.

(c) Solve these problems again but now represent the sine- and cosine-functions by complex-valued exponential functions as derived in the first exercise. Carry out the integrals as you would do for a real-valued exponential function, i.e., the antiderivative of $\exp(\mathrm{i}ax)$ is

$$\frac{1}{ia}\exp(\mathrm{i}ax).$$

2. (Discrete Fourier Transform)

In class, Fourier transform was introduced by sine and cosine series

$$f(t) = \lim_{n\to\infty}\left[\sum_{k=0}^{n} a_k\,\cos(2\pi kt/T) + \sum_{k=0}^{n} b_k\,\sin(2\pi kt/T)\right]\ . \tag{1}$$

Derive complex-valued constants $\hat{f}_k$ as functions of a_k and b_k such that this series can be expressed as

$$f(t) = \frac{1}{T}\lim_{n\to\infty}\sum_{k=-n}^{n}\hat{f}_k\,\mathrm{e}^{2\pi\mathrm{i}kt/T} \tag{2}$$

To do so, use $\mathrm{e}^{ix} = \cos(x) + \mathrm{i}\,\sin(x)$!