

# Statistical Models and Data Analysis

Summer term 2018

## Problem Set 5

14.05.201

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The solutions to this exercise should be ready by 2 pm on 28.5.2018. If you have any questions, please send me an email at stemmler@bio.lmu.de. You may submit your exercises by email in one file, but please be legible!

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1. (Fourier transforms) Compute the Fourier transforms of the following functions:

$$g(x) = \exp\left(-\frac{x^2}{2}\right) \quad (\text{use the fact that } \int_{-\infty}^{\infty} \exp(-x^2) dx = \sqrt{\pi})$$
$$\text{sinc}(x) = \frac{\sin(x)}{x} \quad (\text{rewrite the sine function as the imaginary part of } \exp(ix) )$$

Plot both the original function and its Fourier transform.

2. (Similarity transformation) Given a function  $f(x)$ , we can ask what happens to  $f(x)$  and its Fourier transform  $\mathcal{F}(f(x)) = \tilde{f}(\omega)$  when we rescale the  $x$ -axis by letting  $x \rightarrow ax$  with  $a > 0$ .

Using the definition of the Fourier transform, as shown in class, show that

$$\mathcal{F}(f(ax)) = \frac{1}{a} \tilde{f}\left(\frac{\omega}{a}\right).$$

Use this property and the result from the first exercise to compute the Fourier transform of a Gaussian function  $g(x) = \exp(-x^2/(2\sigma^2))$ . If  $\sigma$  is large (and the Gaussian is broad), what happens to the Fourier transform?

3. (Fourier transform of an amplitude modulated signal)

Consider the function

$$f(t) = [1 + a \sin(\omega_m t)] \sin(\omega_c t)$$

with  $0 \leq a \leq 1$ .

Compute the Fourier-Transform of  $f(t)$ . To do so, express the cosine functions in terms of complex exponentials and use the formula for the Fourier representation of the delta function

$$\delta(y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ixy} dx$$

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