

Statistical Models and Data Analysis

Summer term 2018

Problem Set 8

11.6.2018

Please try your hand at these exercises and foot them to me by 2 pm on 18.6.2018. You can send me email at stemmler@bio.lmu.de if you have questions.

1. (Constrained Optimization) Maximize

$$f(x) = x^2 + y^2$$

subject to the constraints

$$2x + y \leq 2$$

$$x \geq 0$$

$$y \geq 0$$

Use Lagrange multipliers and keep in mind that the constraints are *inequalities*, not equalities.

2. (Mean and Variance of Conditional Data) Suppose the number of visitors to a museum in a given day is N . We have estimates for the mean and variance of N , namely $\mathbb{E}(N) = 400$ and $\text{Var}(N) = 200$. Let Y_i represent the monetary value of souvenirs from the museum store that the i -th person buys. You know, things like t-shirts with the slogans “History, it’s about time” or “May the Mass \times Acceleration be with you”. These sell for outrageous prices, but you feel good about yourself because you are supporting a worthy cause. Let the Y_i ’s be independent of N and also of each other. Furthermore, let us suppose that each person has the same variance and same mean

$$\mathbb{E}Y_i = 20.$$

$$\text{Var}(Y_i) = 625.$$

Note that the standard deviation (square root of the variance) is higher than the mean (some museum visitors buy nothing). Let S be the total sales $S = \sum_{i=1}^N Y_i$.

- Find the values for $\mathbb{E}(S)$ and $\text{Var}(S)$.
- Given your knowledge of statistics, do you think S will be approximately distributed as a Gaussian?
- If S were Gaussian, what is the probability that the sales on any given day will be above 10,000 Euros?