Statistical Models and Data Analysis

Summer term 2018

Problem Set 9 **13.7.2017**

Please submit these exercises by 2 pm on 25.8.2018. You can send me email at stemmler@bio.lmu.de if you have questions.

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1. (Poisson distributions) A neuron spikes every 20 milliseconds on average. The number of spikes in any time period are Poisson distributed. Assume that the neuron has just spiked, and let T be the time to the next spike.

(a) Find the probability density function of T.

(b) Show that the expected number of spikes the neuron emits until it pauses 100 milliseconds or more is

$$e^5 - 1.$$

(c) Determine the expected interspike interval between spikes that are less than a 100 milliseconds apart.

(d) Hence find the expected time until the neuron pauses for 100 milliseconds or more.

2. (More Poisson) Consider a slice of $\Omega = 1\ \text{cm}^2$ of gray matter. Imagine that there are $N = 10,000$ synapses within this area, whose locations in the plane of the slice are completely random (we ignore the vertical z-direction).

(a) Show that the number of synapses within a circular disc-like area $A \subset \Omega$ obeys a Poisson distribution. Find the Poisson rate parameter λ as a function of the synapse density $k = N/\Omega$.

(b) Given a synapse at a particular location (in the interior of the slice, not at the edges), show that the probability distribution of the next closest synapse being at a distance d is a Weibull distribution

$$p(d) \sim 2\pi k d \exp\left(-k\pi d^2\right).$$

(c) What is the expected value of $\langle d \rangle$ as a function of k? (Hint: you might be able to use the fact that $\int_0^\infty \exp(-k\pi r^2)\ dr = \frac{1}{2\sqrt{k}}$)

(d) Bound the probability that the nearest synapse lies more than 1 mm away. Compare the Markov and Chebyshev inequality bounds. For reference, for a non-negative random variable X,

$$P(X \geq a) \leq \frac{\mu}{a} \qquad \text{Markov,}$$

while the following bounds apply to any random variable with a finite mean and variance:

$$P(|X - \mu| > a) \leq \frac{\sigma^2}{a^2} \qquad \text{Chebyshev}$$

$$P(X > \mu + a) \leq \frac{\sigma^2}{\sigma^2 + a^2} \qquad \text{one-sided Chebyshev}$$